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A note on the Probability that first component fails before the second component

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ABSTRACT

The reliability (unreliability) and life testing are important topics in the field of engineering, electronic, medicine, economic and many more, where we are interested in, life of components, human organs, subsystem and system. Statistically, a probability distribution failure time (life time) of a certain form is usually assumed to give reliability of a component for a system for each time t. Some well known parametric life time models ($T \geq 0$) are Exponential, Weibull, Inverse Weibull, Gamma, Lognormal, normal (T>0 ; left truncated) etc.

In this paper we consider a system that, has two components with independent but non-identical life time probabilities explained by two distinct random variables say T_1 and T_2 , where T_1 has a constant hazard rate and T_2 has an increasing hazard respectively . For this system, we derive, evaluate and simulate the probability component 1(be a better component) fails before 2 which is given by

$$P[T_2 > T_1] = \int_0^\infty f_1(t) R_2(t) dt$$

where $f_1(t)$ is the failure probability density function of life time of component 1

and $R_2(t)$ is the reliability function of component 2.

Keywords: Probability density function, probability of failure, reliability and hazard rate function.

1. RELIABILITY STATISTICAL APPROACH

The reliability or survival time of a component or a system is measured by using laws of mathematical statistics. That is, random variable, failure time has probability distribution of a certain from is assumed to give the reliability of a component or a system each time t. The reliability or failure probability varies with time. To monitor the life time of a unit across the support of its life time distribution, the hazard rate $h(t)$or failure rate or force of mortality is used. In fact, the hazard rate is more informative about underling mechanism of failure than the other representatives of life time distribution. The hazard rate, (instantaneous rate of failure, rate in terms of probability) is the ratio of probability distribution function $f(t)$ to the reliability function $R(t)$

$$h(t) = \frac{f(t)}{R(t)} \qquad (1.1)$$

$$h(t) = \frac{f(t)}{1-F(t)} \qquad (1.2)$$

If the probability density function of T is completely known, one may be able to determine the hazard rate. Conversely, if $h(t)$ is known then we can obtained the density of the random variable T.

Since f(t) may be expressed as
f(t) : probability density function (pdf) of failure time;
F(t)=P(T≤ t) : is the failure distribution function;
R (t)=1-F(t) : is the probability that the component has not failed by time t.

$$f(t) = F^{'}(t) = -R^{'}(t)$$

$$= h(t)R(t)e^{-\int_0^t h(x)dx} \qquad (1.3)$$

It is interesting to note that, these representatives are interrelated, if anyone is known others can be find.
In this paper we consider a series system of two independent non-identical components with different hazard rates.
a) We assume that one component has constant hazard rate (no aging) i.e. independent of time Crowder et al.(1991) , no wear tear and no improvement, and
b) one component has increasing hazard rate aging with time i.e. wear out with time.
The component with constant hazard rate is supposed to be better component long lasting, because failure occurs at random.
These properties of hazard function are held by two well known failure time distributions exponential distribution and normal distribution respectively. These distributions are the basic models exemplified for number of theoretical concepts in reliability studies.
Assume that the random variable T_1 and T_2 represent the life times of two components. The components can be considered as the independent and non identical. Comparing their reliability using Brown and Rtuermiller (1973) and LifeDataWeb http://www.weibul.com/ approach evaluating the quantity $P(T_2 > T1$, that is component having life time $T2$ survives the longest.

2. TIME TO FAILURE DISTRIBUTION (LIFE TIME DISTRIBUTION)

This section considers probability distributions which are most often used in reliability as time to failure distributions. These distributions include the distributions which have well known probability density functions for describing time to failure in many situations.

2.1 Exponential distribution

One of the best known, most useful and most thoroughly explored failure distribution is the exponential. This failure distribution is applicable to many types of component (system) failure, specially in electrical system.

The probability density function of commonly used exponential distribution can be obtained from constant hazard rate, λ then

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$$f(t) = \lambda e^{-\lambda t} \quad \text{for } t \geq 0 , \lambda > 0 \qquad (2.1)$$

Where λ= failure rate per unit time and t is time of failure

The reliability function for exponential distribution is

$$R(t) = \int_{x}^{\infty} \lambda e^{-\lambda t} dt$$

$$R(t) = e^{-\lambda t} \quad ; t \geq 0 \qquad (2.2)$$

The mean time to failure (MTTF) is

$$MTTF = \mu = \int_{0}^{\infty} R(t)\, dt$$

$$MTTF = 1/\lambda > 0 \qquad (2.3)$$

The hazard rate function is using (2.2)

$$h(t) = \lambda \qquad (2.4)$$

It shows the time of failure is a constant.

Following Figure 1.1 shows their graphic illustration of exponential pdf, reliability function and hazard function

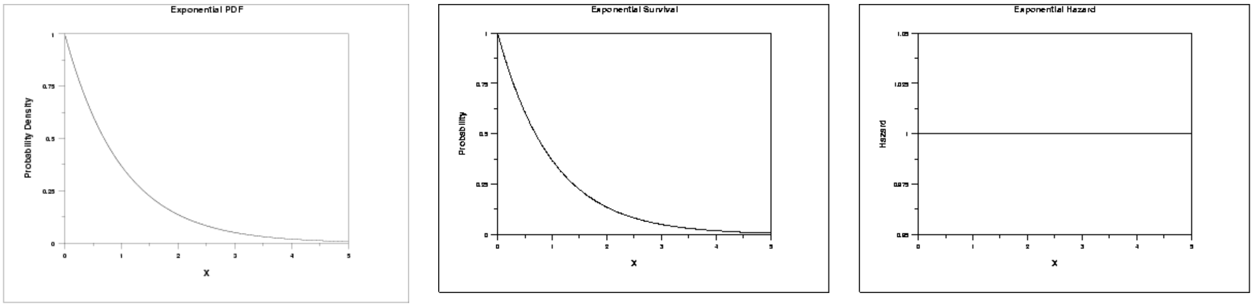

Figure 1.1.

2.2 Normal distribution

In this paper Normal distribution is used to model the reliability for component experience wear out failures. Given a mean life μ, and standard deviation, σ, the reliability can be determined at a specific point time t. For the normal distribution (probability density function), reliability function and hazard rate are given by

$$f(t) = \frac{1}{o\sqrt{2\pi}} e^{\frac{-1}{2}\frac{(t-\mu)^2}{\sigma^2}} \text{-}\infty < t < \infty \qquad (2.5)$$

where μ is the mean and σ is the standard deviation.

The reliability function for normal distribution is

$$R(t) = 1 - \Phi\left(\frac{t-\mu}{\sigma}\right) \qquad (2.6)$$

provided $\left(\frac{\mu}{\sigma}\right) > 3$, Kishor (2001). where $\left(\frac{t-\mu}{\sigma}\right) = \int_{-\infty}^{\frac{t-\mu}{\sigma}} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$, is the standard normal distribution function .

Also note that

$$MTTF = \mu \qquad (2.7)$$

$$h(t) = \frac{f(t)}{R(t)} \qquad (2.8)$$

The pdf, reliability function and hazard function of the normal distribution are appeared in Figure 2.2.

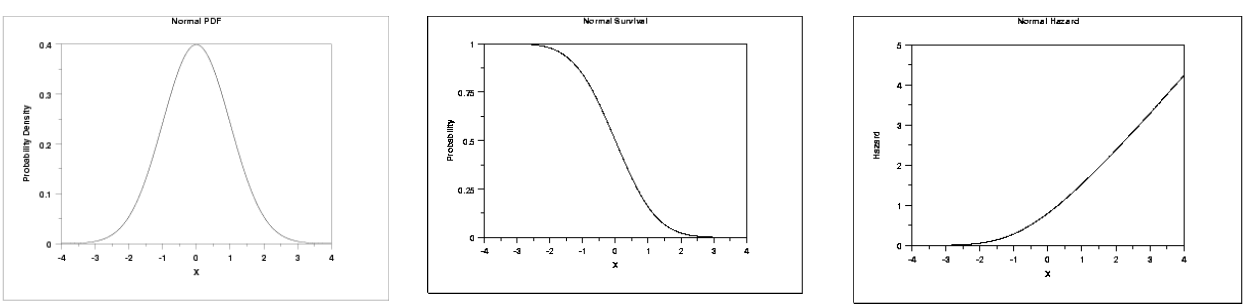

Figure 2.2.

3. EVALUATION OF $P(T_2 > T_1)$

The expression $\boldsymbol{P(T_2 > T_1)}$is formulated as

The joint probability density function of T_1 and T_2 is

$$f(t_1, t_2) = f(t_1)f(t_2)$$

If T_1 and T_2 are independent.

$$P(T_2 > T_1) = \iint_{t_2=t_1}^{\infty} f_1\,(t_1)(t_2)dt_2dt_1 \qquad (3.1)$$

$$= \int_0^{\infty} f(t_1)\,[1 - F_2(t_1)]dt_1$$

$$= \int_0^{\infty} f(t_1)\,R_2(t_1)dt_1$$

$$R(t) = \int_0^{\infty} f(t)\,R_2(t)dt \qquad (3.2)$$

In this section we define concept of probability where T_2 is greater than T_1 extending the concept of probability ordering given by Mi (1999). In other words, if for all values of t,

$$R(t) = P(T_2 > T_1)$$

$R(t) > 0.5$, we say that the system with life time T_2 is better.

$R(t) = 0.5$, the components are identical and independent of time.

$R(t) < 0.5$, the system with life time T_1 is better than the system with life time T_2.

Zio et al.(2011) have found $P(T_2 > T_1)$ when both T_1 and T_2 follow exponentially distributed failure times.

Let component number 1 follows normal distribution with incensing failure rate (IFR)

$$f_1(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-1}{2}\frac{(t-\mu)^2}{\sigma^2}}$$

And component number 2 follows exponential distribution with reliability function

$$R_2(t) = e^{-\lambda t}$$

The reliability is determined by (3.2)

$$P[T_2 > T_1] = \int_0^{\infty} f_1(t)R_2(t)dt$$

After doing tedious mathematics expression (3.3) is obtained

$$P[T_2 > T_1] = 1 - \{Exp\left[\frac{-1}{2}(2\lambda\mu - \lambda^2\sigma^2)\right]\Phi\left(\frac{\mu-\lambda\sigma^2}{\sigma}\right)\} \qquad \text{provided } \frac{\mu}{\sigma} > 3 \qquad (3.3)$$

$\Phi(.)$ is standard normal distribution function .

For some selected parameters we evaluate this reliability in table 1.

Let normal failure model has mean time to failure (MTTF) i.e. μ =1000 hours with standard deviation σ (100, 200, 300) hours. While exponential model has MTTF =$MTTF = \frac{1}{\lambda}$=1000 hours.

σ	MTTF=λ=μ=1000
	$P(T_2 > T_1)$
100	0.8619
200	0.8535
300	0.8388

Table 1. computed values of $P(T_2 > T_1)$.

In table1, it is obvious that, component 2 with constant hazard rate survives the longest since $P(T_2 > T_1)$>0.5 in each case.

4. APPLICATION (DATA ANALYSIS)

Because the real data is difficult to achieve due to commercial considerations, a well known alternative is Monte Carlo (MTC) simulation technique. For application purposes, consider a system with two components, where $P(T_2 > T_1)$ are determined by Monte Carlo (MTC) simulation, which is a method used to confirm analytical results. In this paper random observations obtained directly from Minitab. Several Mote Carlo simulations were performed. In each simulation, one hundred distinct random samples each consisting of twenty $(T_{1,}T_2)$ pairs were drawn from normal distribution and exponential distribution for selected parameters. The twenty pairs of observation utilized to estimate$P(T_2 > T_1)$. Results are shown in Table 2. For the purpose of comparison we keep MTTF =1000 in both the models. The combinations of parameters are used (μ=1000, σ=100, λ=1000), (μ=1000, σ=200, λ=1000) and (μ=1000, σ=300, λ=1000).

σ	MTTF=λ=μ=1000
	$P(T_2 > T_1)$
100	0.8439
200	0.8121
300	0.7946

Table2. Summary results for 100 simulations of $P(T_2 > T_1)$.

The results in Table 2 reveal that there in no much difference between analytical and sample results.

5. CONCLUSION

If a system is designed, with two components in series component 1 that has increasing hazard rate and second component 2 has constantan hard rate then system performance will be long lasting.

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